SECTION I

Overview

This paper merely scratches the surface of the subject of Optimal Control, which is a large and growing area of research. What is hoped is that an appreciation will be given of the value of pursuing this type of research at the U.W.I. In this context, some basic motivations for the study of optimal control are necessary. Firstly, for any management situation, be it of the economy, or of a microeconomic system, such as, a business enterprise, the thing that is being managed is a set of scarce resources. These resources generally tend to change in value over time, or are used up over time. Given this situation of change, it is useful to have a systematic approach to optimize (maximize) the benefits to be derived from these resources over some specified period of time. Thus, the function which tells us how the resources should optimally be used to satisfy some value criteria (social welfare maximization, profit maximization, etc.) is necessary. Optimal control theory embodies techniques which allow us to determine this function.

The foregoing provides a motivation for the study of optimal control based on the need to achieve proper resource allocation over time. Another motivating factor, particularly in the management of the macro economy, is the examination of different policy alternatives; for example, government may want to know whether providing a subsidy to the manufacturing sector in the form of lower interest rates is better than using higher import duties to protect the sector. In this type of setting what can be done is the examination of the effects on gross domestic product over time of the subsidy, and with the tariff in place. The results of these two policies can then be compared to see which yields a higher rate of growth of the gross domestic product, and this alternative would be chosen as the policy. While this example is a specific one relating to tariff versus subsidy, this is illustrative of the general sort of economic policy problems which governments face on an ongoing basis.

It must be pointed out at the onset that the study of optimal control requires mathematical rigour at the theoretical stage of analysis, and where empirical tests are performed on the results of the theoretical models a high level of statistical competence is also required. Therefore, to incorporate this branch of economics into the curriculum of the U.W.I. economics programme requires students that have a strong foundation in mathematics. It is hoped that these types of students can be drawn into this interesting field of optimal control because of the many and varied real world application that exist. In the physical sciences for example, the field of servo-mechanics has its roots in optimal control theory.

Now a brief word about the organisation of this paper. Section II provides a heuristic solution to the optimal control problem, showing how investment in human and physical capital should made over time to maximize social welfare. This section utilises the methodology of the "maximum principle" to obtain a solution to the problem. With this method, a set of differential equations (preferably linear, and first order) are generated, which are then solved to yield the functional forms of the evolution of the resources over time. Section III provides some hints on how to use the solutions to analyse policy issues, and how to test for the validity of the models (i.e. the economic implications of the solutions).

NOMENCLATURE

α_i: Share of consumption expenditure devoted to good i.

β': Physical capital's relative share in good L output.

β: Physical capital's relative share in good H output.

δ_i: Maximum rate of learning in sector i.

ϵ: Parameter in physical capital growth rate in small country.

ξ_i: Fraction of labor force in sector i.

ϕ_i': Good i consumption growth rate in free trade.

ϕ_i: Good i consumption growth rate in autarky.

γ: Externality exponent.

λ: Population growth rate.

η: Relative investment parameter in small country.

π: Rate of time preference.

ν_i: Growth rate of human capital in sector i.

θ_j: Shadow price of capital j.

ζ: A parameter reflecting the ratio of domestic/foreign capital.

ζ': Ratio of world consumption of good H to that in the "big" country, B.

φ: Fraction of country B's consumption of L out of its profits earned abroad.

A_i: Technical change parameter in sector i.

a: Parameter in physical capital growth rate in the "small" country.

b: Parameter in GNP growth rate in small country.

h_i: Human capital, or skill level, in sector i.

h_{ai}: Average level of human capital in sector i (an external effect).

u_i: Fraction of time spent by labor in good i production.

N: Population.

K_i: Physical capital employed in good i production.

k_i: Per capita physical capital in good i production.

r_i: Per capita real GNP (output) growth rate in sector i.

r: Economy-wide real GNP growth rate.

p: Relative price of good H in terms of good L.

c_i: Per capita consumption of good i.

u: Individual utility associated with consumption of goods H and L.

S: Denotes "small" country, or LDC.

B: Denotes "big" country.

R: Ratio of domestic/foreign capital in country S.

THEORETICAL PROCEDURE

This paper uses a dynamic model to analyze growth in which the engine of growth, human capital, is endogenous to the model. In the model the average level of human capital imposes a positive externality on the production process. This causes the optimum and equilibrium rates of growth to diverge, with the expected result that free market forces alone will not produce an optimum level of human capital in the economy. Both the optimum and equilibrium growth rates of all relevant variables, and their corresponding levels through time are explicitly determined. These solutions are facilitated by the use of simplifying assumptions on the functional forms of the production and social welfare functions (log-linear utility functions, and a Cobb-Douglas type of production function).

Other simplifications include assuming away heterogeneity in preferences and in the types of a given good. This allows the analysis to take place in the familiar setting of a typical agent, with two sectors represented by aggregate production functions. Also, to capture the fact that there is strong evidence of the dominance of a single homogenous good in the economy of many LDC's (see table 2) a stylized open economy model of specialization in production is analysed.

A final simplification is the that of a constant terms of trade. The constant terms of trade is a useful device for showing the role of exports in economic growth in the LDC (the recipient of the FDI), since with specialization in production, and constant terms of trade the rate of exports growth is the same as the growth rate of consumption of the domestically produced good. The reason for this is that the rates of growth of consumption of the two goods must be the same in order for relative prices (terms of trade) to be constant in free trade. The full list of assumptions follows.

I.2: Autarkic Analysis (Both Countries)

The purpose of this section is to determine the functional form of the balanced Gross Domestic Product growth rate, and to determine if there is a difference between the growth rates in the two countries in the absence of trade.

The problem faced by each country is the maximization of present-value welfare, as measured by a discounted utility index, subject to restrictions on the mechanisms for the formation of human and physical capital. Formally, the problem is:

(1) $\text{Max} \int_0^\infty u(c_L,c_H)e^{-\pi t}N(t)dt$

Where, $u(c_L,c_H) = \alpha_L \log(c_L) + \alpha_H \log(c_H)$

Subject to:

(2) $N(t)c_L(t) + \dot{K}_L(t) = A_L K_L(t)^{\beta'}[u_L(t)h_L(t)N(t)\xi_L]^{1-\beta'}h_{aL}(t)^\gamma$

(3) $N(t)c_H(t) + \dot{K}_H(t) = A_H K_H(t)^{\beta}[u_H(t)h_H(t)N(t)\xi_H]^{1-\beta}h_{aH}(t)^\gamma$

(4) $\dot{h}_L(t) = h_L(t)\delta_L[1-u_L(t)]$

(5) $\dot{h}_H(t) = h_H(t)\delta_H[1-u_H(t)]$

In equation (5) $\xi_L + \xi_H = 1$

ξ_i is the fraction of the labor force in sector i.[3]

Therefore, $[u_i(t)h_i(t)N(t)\xi_i]$ is the effective labor force in sector i. Also, The external effect of human capital is the average skill level, h_{ai}, in each sector, and is defined, as in Lucas (1988), as:

(6) $h_{ai} = [\int_0^\infty h_i N(h_i)dh_i]/[\int_0^\infty N(h_i)dh_i]$

Where, $N(h_i)$ is the number of workers in sector i with skill level h_i. In the rest of this paper, as in Lucas, workers are identical, so $h_{ai} = h_i$. However the designation h_{ai} will be retained to emphasize the externalities associated with human capital.

The current value Hamiltonian, $H(K_L,K_H,\theta_L,\theta_H,\theta_1,\theta_2,c_L,c_H,h_L,h_H,u_L,u_H,t)$ is:

(7) $$H = N[\alpha_L \log(c_L) + \alpha_H \log(c_H)] + \theta_L[A_L K_L^{\beta'}(u_L N h_L \xi_L)^{1-\beta'} h_{aL}^{\gamma} - Nc_L] + \theta_H[A_H K_H^{\beta}(u_H N h_H \xi_H)^{1-\beta} h_{aH}^{\gamma} - Nc_H] + \theta_1[\delta_L h_L(1-u_L)] + \theta_2[\delta_H h_H(1-u_H)];$$

Where, θ_i is the shadow price of capital in sector i. marginal value of capital

$\theta_i = \partial V/\partial s_i$, where $V \equiv \int(\cdot)$

3. In the usual case ξ_i is determined from the result of each agent selecting the sector with the highest present value of wages net of education cost. However, since all agents are assumed alike they will make the same choice. Using the method of Grossman and Helpman (1991, chp. 5) to overcome this problem, each type of labor assumed essential.

First Order Conditions

(8) $\delta H/\delta c_L = 0$: $N\alpha_L/c_L = N\theta_L$. This implies $\theta_L = \alpha_L/c_L$.

(9) $\delta H/\delta c_H = 0$: $\theta_H = \alpha_H/c_H$ (By symmetry).

(10) $\delta H/\delta u_L = 0 : \theta_L(1-\beta')A_L K_L^{\beta'}(u_L N h_L \xi_L)^{-\beta'} N h_L^{1+\gamma} = \theta_1 \delta_L h_L$

(11) $\delta H/\delta u_H = 0 : \theta_H(1-\beta)A_H K_H^{\beta}(u_H N h_H \xi_H)^{-\beta} N h_H^{1+\gamma} = \theta_2 \delta_H h_H$

Interpretation of FOC's:

(8) & (9): Goods are equally valuable at the margin in their two uses as capital and consumables, i.e. marginal value of physical capital equals marginal utility of the corresponding consumption good in each time period.

(10) & (11): Time equally valuable at the margin in its two uses as productive labor or schooling, i.e. human capital formation.

Optimal Capital Allocations (From Optimum Price Evolutions)

Since each worker is identical, then:

(12a) $h_i(t) = h_{ai}(t)$

By Pontryagin's Maximum Principle, optimum capital allocation are obtained from the optimum price evolution as follows:

(12b) $S_i: \dot{\theta}_i = \pi\theta_i - \delta H/\delta s_i, \quad s_i \in (K_L, K_H, h_L, h_H)$

(13) $K_L: \dot{\theta}_L = \pi\theta_L - \theta_L\beta' A_L K_L^{\beta'-1}(u_L N h_L \xi_L)^{1-\beta'} h_{aL}^{\gamma}$

(14) $K_H: \dot{\theta}_H = \pi\theta_H - \theta_H\beta A_H K_H^{\beta-1}(u_H N h_H \xi_H)^{1-\beta} h_{aH}^{\gamma}$

(15) $h_L: \dot{\theta}_1 = \pi\theta_1 - \theta_L(1-\beta'+\gamma) A_L K_L^{\beta'}(u_L N \xi_L)^{1-\beta'} h_L^{\gamma-\beta'} - \theta_1\delta_L(1-u_L)$

(16) $h_H: \dot{\theta}_2 = \pi\theta_2 - \theta_H(1-\beta+\gamma) A_H K_H^{\beta}(u_H N \xi_H)^{1-\beta} h_H^{\gamma-\beta} - \theta_2\delta_H(1-u_H)$

Equilibrium Price Evolution

In equilibrium each agent takes the external value of human capital h_{ai} as given in his sector. Thus, eqns. (15) and (16) are modified as follows:

$$(17)\quad \dot{\theta}_1 = \pi\theta_1 - \theta_L(1-\beta')A_L K_L^{\beta'}(u_L N\xi_L)^{1-\beta'}h_L^{-\beta'}h_{aL}^{\gamma} - \theta_1\delta_L(1-u_L)$$

Now in equilibrium, $h_L = h_{aL}$. Thus,

$$(18)\quad \dot{\theta}_1 = \pi\theta_1 - \theta_L(1-\beta')A_L K_L^{\beta'}(u_L N\xi_L)^{1-\beta'}h_L^{\gamma-\beta'} - \theta_1\delta_L(1-u_L)$$

Similarly in sector H,

$$(19)\quad \dot{\theta}_2 = \pi\theta_2 - \theta_H(1-\beta)A_H K_H^{\beta}(u_H N\xi_H)^{1-\beta}h_H^{\gamma-\beta} - \theta_2\delta_H(1-u_H)$$

Transversality Conditions

The following transversality conditions reflect the fact that all capital must become progressively less valuable, so that at the infinite horizon the value of all capital stocks become zero. This concept is easier to grasp in the case of a finite horizon problem in which some amount of goods in the form of capital will be left unconsumed at the end of the finite time period, T, if the value of capital at T exceeds that at T+1. In such a situation the agent would not be maximizing utility over the planning period T. Thus, the transversality conditions are statements about the terminal values of each type of capital which satisfy the requirements for optimality.[4]

$$(20)\quad K_L:\ \lim_{t\to\infty} e^{-\pi t}\theta_L(t)K_L(t) = 0$$

$$(21)\quad K_H:\ \lim_{t\to\infty} e^{-\pi t}\theta_H(t)K_H(t) = 0$$

$$(22)\quad h_L:\ \lim_{t\to\infty} e^{-\pi t}\theta_1(t)h_L(t) = 0$$

$$(23)\quad h_H:\ \lim_{t\to\infty} e^{-\pi t}\theta_2(t)h_H(t) = 0$$

[4]. To guarantee that (20)- (23) hold the terminal values of K_L and K_H must equal zero, and $\lim_{t\to\infty} \theta_i(t) = 0$ (i=1,2). Silberberg(1978, ch. 18), and Petit (1990, chp. 5) provide good discussions of these concepts.

Optimal And Equilibrium Growth Paths

The optimum growth paths for each type of capital ($K_L, K_H, h_L,$ and h_H) and the corresponding prices are described in eqns. (2)-(5), and eqns. (13)-(16), respectively. The equilibrium growth paths for these variables are described by eqns. (2)-(5), and eqns. (13), (14), (18), (and (19). To characterize these paths, consider optimal and equilibrium growths along balanced paths as in Lucas(1988), i.e. capital and consumption growing at constant percentage rates and prices diminishing at constant rates, with $u_i(t)$ being a determined constant. Let the balanced growth rate of consumption of each good be defined as ϕ_L and ϕ_H for goods L and H, respectively. Thus,

$$\dot{c}_L/c_L = \phi_L, \text{ and } \dot{c}_H/c_H = \phi_H.$$

Then, from (8):

(24a) $\dot{\theta}_L/\theta_L = -[\alpha_L(\dot{c}_L/c_L^2)]/[\alpha_L/c_L] = -\dot{c}_L/c_L = -\phi_L$

Likewise, from (9):

(24b) $\dot{\theta}_H/\theta_H = -\phi_H$

Per Capita Capital Growth Rates ($\dot{k}_i/k_i$)

Consider sector L first. Examining (13), and suppressing time notation, t:

% Percentage change in marginal value of capital

$$\text{(25)}\quad \dot{\theta}_L/\theta_L = \pi - \beta' A_L K_L^{\beta'-1}(u_L N h_L \xi_L)^{1-\beta'} h_{aL}^{\gamma} = -\phi_L$$

Thus,

$$\text{(26)}\quad \beta' A_L K_L^{\beta'-1}(u_L N h_L \xi_L)^{1-\beta'} h_{aL}^{\gamma} = \phi_L + \pi$$

Eqn. (26) says that the marginal product of physical capital in sector L on the balanced growth path is equal to the sum of the per capita growth rate in consumption and the discount rate, π. This is an important relationship, because it guarantees that there will be further investment of capital in sector L as MPK_L exceeds the required rate of return (equal to π). Recall from static analysis that investment takes place up to the point where $MPK = \pi$. In the current setting only when $\phi_L = 0$ will MPK_L be equal to π. This occurs when investment in schooling falls to zero, i.e. at the infinite horizon (see eqn. 3 section I.3).[5] Grossman and Helpman(1991, chp.1) provide a useful discussion of this concept, contrasting endogenous growth models of this type with the neoclassical/Solow exogenous growth models.

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[5]. In the Technical Note following eqn. 3 in Appendix A.2 it s shown that a boundary condition on the value of the external arameter γ is necessary to assure long run growth.

If eqn. (2) is divided by K_L:

(27) $Nc_L/K_L + \dot{K}_L/K_L = A_L K_L^{\beta'-1}(u_L N h_{L\ L})^{1-\beta'} h_{aL}^{\gamma} = (\phi + \pi)/\beta'$

Since $\dot{K}_L/K_L$ is constant on the balanced path, eqn. (27) also guarantees that Nc_L/K_L is also a constant. Using this fact $\dot{K}_L/K_L$ can be obtained by differentiating the following equation w.r.t. time:

$Nc_L/K_L = \text{constant}$

The result is:

$\dot{N}c_L/K_L + N\dot{c}_L/K_L - (Nc_L/K_L)(\dot{K}_L/K_L) = 0$

Now dividing by Nc_L/K_L gives:

$\dot{N}/N + \dot{c}_L/c_L - \dot{K}_L/K_L = 0$

Which is the same as,

$\lambda + \phi_L - \dot{K}_L/K_L = 0$

Thus,

(28) $\dot{K}_L/K_L = \phi_L + \lambda$

Now, $k_L = K_L/N$

This implies, $K_L = k_L N$

And, $\dot{K}_L = dK_L/dt = d(k_L N)/dt = \dot{k}_L N + k_L \dot{N}$

Thus, from (28):

$$(\dot{k}_L N + k_L \dot{N})/k_L N = \phi_L + \lambda$$

So, $\dot{k}_L/k_L + \dot{N}/N = \phi_L + \lambda$

And since $\dot{N}/N = \lambda$

(29) $\dot{k}_L/k_L = \phi_L$

Therefore, per capita capital in sector L, and per capita consumption of goods produced in this sector both grow at the same constant rate ϕ_L along the balanced paths. The optimum and equilibrium growth paths are analysed separately in appendix A.2.

Due to the similarity in the technological constraints in the two sectors, the value for $\dot{k}_H/k_H$ can be written as:

(30) $\dot{k}_H/k_H = \phi_H$

<u>Balanced Real GDP Growth</u>

Following the previous procedure, start by analyzing sector L, and suppress the time notation:

Examining (2), the Real GDP (RGDP) growth rate in sector L, r_L, is:

$$r_L = \{d/dt[\, Nc_L + \dot{K}_L]\}/\{Nc_L + \dot{K}_L\}$$

i.e.:

$$r_L = (\dot{N}c_L + N\dot{c}_L + \ddot{K}_L)/(Nc_L + \dot{K}_L) \;\; ; \text{ where, } \ddot{K}_L = d/dt(\dot{K}_L)$$

With the definition of $\ddot{K}_L$, it is obvious from eqn. (28) that:

$$\ddot{K}_L = (\phi_L + \lambda)\dot{K}_L$$

Using this value for $\ddot{K}_L$, and dividing both the numerator and denominator of the second expression of r_L by Nc_L results in:

(31) $r_L = [\lambda + \phi_L + (\phi_L+\lambda)(\dot{K}_L/Nc_L)]/[1 + (\dot{K}_L/Nc_L)]$

Factoring out $(\lambda + \phi_L)$ gives:

(32) $r_L = \lambda + \phi_L$

RGDP Growth Continued

RGDP growth rate in sector H is obtained via a similar procedure as that for sector L, and is:

$$(33) \quad r_H = \lambda + \phi_H$$

The economy-wide RGDP growth rate, r, may be taken as a geometric average across the two sectors. Thus,

$$(34) \quad r = (\lambda + \phi_L)^{1/A}(\lambda + \phi_H)^{1/B}$$

Where, $1/A + 1/B = 1$

Now, the rate of consumption of each good is constant on a balanced path since P_H/P_L is assumed constant. Thus, the growth rate of the economy can be expressed in terms of any one of the sectoral growth rates:

$$(35) \quad r = \lambda + \phi_L = \lambda + \phi_H = \lambda + \phi$$

Thus, for the autarkic economy output and capital grow at the same rate.

Comparison Of Real GDP Growth Rates In Autarky

In section I.3 which follows it is shown in equation (5) that real GDP growth rate is related to the technology parameters in the following manner:

(36) $r = \lambda + [(1 - \beta + \gamma)/(1 - \beta)]\upsilon$

(the sector-L notation is dropped for convenience)

Substituting the optimum value of υ from equation (3) Appendix A.2 in (36) gives:[6]

(37) $r = \lambda + \delta(1-\beta+\gamma)/(1-\beta) -_{\gamma}\pi + \lambda$

$= 2\lambda + \delta(1-\beta+\gamma)/(1-\beta) - \pi$

6. The optimum value for υ from eqn. 3 Appendix A.2 is:
$\upsilon^*_L = \delta_L - [(1-\beta')(\pi-\lambda)]/(1-\beta'+\gamma)$

From equation (37) it can be seen that the size of a country, i.e. its initial endowment of physical capital, is not relevant for economic growth in this model. Rather, the model predicts that what would make one country grow faster in autarky than the other would be one or more of the following:

(i) Higher population growth rate, λ.

(ii) Higher maximum feasible human capital growth rate, δ.

(iii) Higher capital share of output, β.

(iv) Lower real interest rate, π.

II.1: Overview Of Regression And Data

i). Empirical Analysis Overview

In this chapter regression analysis will be done to test the predictions of the model of economic growth for the "small" trading country. Recall from section I.4, equation (28), that the structural form of the long run growth rate of gross national product (GNP), r_s, is:

$$(1)\quad r_s = b\phi'_{LS} + b\lambda_s; \quad b<1.$$

Where the growth rate in consumption, ϕ'_{LS}, was shown in section I.3, equation (3), to be related to human capital and the parameters of taste, and technology in the following manner:

$$(2)\quad \phi'_{LS} = [(1 + \gamma - \beta')/(1 - \beta')]\upsilon_{LS}$$

In Appendix A.2 the relationships between human capital growth rate υ_{LS} and the parameters of human capital formation, taste, and technology are shown. Using the simpler expression of the equilibrium growth path, equation (4), in equation (2) above, and substituting the results in equation (1) gives:

$$(3)\quad r_s = b\lambda_s + b[(1 + \gamma - \beta')/(1 - \beta')](\delta_{LS} + \lambda_s - \pi_s)$$

Where, $0 < \beta' < 1$, and $\gamma > 0$.

Equation (3) is the reduced form of the model. The testable hypotheses implicit from this model are the following:

i) Human capital growth rate, δ_{LS}, and population growth rate, λ_S, positively influence real total GNP growth rate, r_S. Population in this model is synonymous with the labor force, since there is no unemployment.

ii) The real interest rate, π_S negatively influences real total GNP growth rate, r_S.

iii) The level of foreign investment has no significant influence on real total GNP growth rate, r_S. However, exogenous increases in foreign capital is predicted to increase r_S (ref. the theoretical result in Appendix A.3). Thus, the growth rate of the foreign capital stock is predicted to positively influence the growth rate of GNP.

iv) Knowledge spillover from abroad (γ) has a positive influence on real total GNP growth rate, r_S. γ will be operationalized in two forms. First, as exports of goods, since it is argued by some researchers that through the export activity new techniques and management skills are learned, usually costlessly, due to the contact of exporters with foreign producers in the same industry (Feder 1982). Second, spillovers will be operationalized as foreign direct investment, since it is also argued that in addition to providing external capital, new management techniques, new forms of business organizations, and knowhow are transferred sometimes costlessly to the foreign affiliate (see Ruffin 1984). Thus, Foreign Direct Investment will serve the dual purpose of capturing the effect of foreign investment on growth, and indicating whether there is a positive externality associated with it. The following criteria is used to indicate whether an externality is associated with either exports or foreign direct investment (FDI):

Exports: If the coefficient is positive, significant, and is greater than would be indicated from the average of the ratio of exports/GDP, then a positive externality is indicated. Thus, if this average were 0.1, then an externality would be indicated if the coefficient on the growth rate of exports exceeded 0.1.

FDI: If the coefficient is positive, and significant, this is an indication that it may have a positive external effect on the economy of the host country. The issue of explicitly testing this hypothesis is discussed at the end of part 2 of this chapter.

From the foregoing overview, the following linear regression model is hypothesized:

$$GDP(t) = \beta_0 + \beta_1 POP(t) + \beta_2 CAP(t) + \beta_3 PRI(t) + \beta_4 RINT(t) + \beta_5 FDI(t) + \beta_6 XP(t) + u(t) \quad \ldots\ldots\ldots\ldots (4)$$

Where,

GDP(t) = Growth rate of GDP (total).

POP(t) = Growth rate of population (proxy for labor force).

CAP(t) = Ratio of domestic investment to GDP.

PRI(t) = Primary school enrollment growth rate (proxy for human capital growth rate).

RINT(t) = Real interest rate.[10]

FDI(t) = Growth rate of foreign capital stock due to the flow of net foreign direct investment.

XP(t) = Growth rate of exported goods.

(All variables are in real, current values)

The hypotheses to be tested in the form of the significance level of the studentized t-statistics of the β_i parameters are:

β_0[11] $= 0$, $\beta_1>0$, $\beta_2>0$, $\beta_3>0$, $\beta_4<0$, $\beta_5>0$, and $\beta_6>0$

10. Some researchers have used the difference between the 3-onth LIBOR (London Interbank Offer Rate) on deposits of $US and he rate of inflation in the country's export prices as a proxy for he real interest rate (Edwards 1986 is one example of this).

11. This hypothesis is the prediction of equation (3), which tates that there is no constant underlying rate of growth which is ndependent of economic factors.

Table 7: Results For Continent Groups

AFRICA (n=17)

Variable	Coefficient	t-Statistic	Significance
CONSTANT	-1.21	-0.40	0.69
POP	0.81	0.84	0.40
CAP	0.11	1.74	0.08
PRI	0.04	1.75	0.08
RINT	-0.05	-0.08	0.43
FDI	0.06	1.77	0.07
XP	0.04	3.23	0.0001

$R^2 = 0.67$, $\rho = 0.75$
(17.5)

AMERICAS (n=21)

Variable	Coefficient	t-Statistic	Significance
CONSTANT	-8.42	-3.00	0.003
POP	1.18	1.35	0.18
CAP	0.33	4.02	0.00008
RATIO	0.08	1.47	0.14
RINT	-0.02	-0.56	0.57
FDI*	0.04	1.45	0.15
XP	0.05	3.53	0.0005

$R^2 = 0.67$, $\rho = 0.77$
(17.6)

ASIA (n=15)

Variable	Coefficient	t-Statistic	Significance
CONSTANT	-1.96	-0.63	0.53
POP	0.04	0.07	0.95
CAP	0.14	1.94	0.05
RATIO	0.10	1.66	0.10
RINT	0.017	0.31	0.76
FDI*	0.046	1.19	0.23
XP	0.087	4.59	0.000008

$R^2 = 0.70$, $\rho = 0.76$
(14.0)

* When RATIO was removed from the regression FDI was found to be significant (p=0.07). This means that the estimator of FDI may be fragile with respect to the conditioning set. This is similar to the findings of Levine and Rennelt (1992).

A Note On Testing Dynamic Relations

Testing dynamic relations is inherently an analysis of time dependent variables; i.e. it is a study of time series, as opposed to cross-sectional analysis. As with all time series analysis phenomena such as autocorrelation, trend, seasonality, and heteroscedasticity acutely affect the estimations as they act to produce biased results, and invalidate the inference procedures of simple estimation techniques, such as Ordinary Least Squares (OLS). To correct for most of these problems use Maximum Likelihood Estimation. The table below shows techniques which can be used for specific problems in the data:

Specific Problem	Method To Correct For Problem
Heteroscedasticity	Auto Regressive Conditional Heterosecdasticity (ARCH)
Autocorrelation (of order 1)	AR-1 Methods: Hildreth-Lu, Cochrane-Orcutt
Trend	Differencing of data, then apply OLS
Data fracture	Average data over disctete intervals, then apply OLS.